TECHNICAL SCIENCES

CATHODE LENS WITH ROTATIONAL AND SEXTUPLE FIELD COMPONENTS

Ibrayev A.
Doctor of Physical and Mathematical Sciences, Professor
KazNU named after Al-Farabi
Baidullayev T.
Master's degree
KazNU named after Al-Farabi
DOI: 10.24412/2701-8369-2021-5-75-77

Abstract
The paper deals with the problems of correction of certain types of aberrations of focusing electronic lenses and electro-optical assemblies using additional elements containing a sextupole component of the electrostatic potential distribution. Aberration correctors make it possible to significantly improve the technical characteristics of a wide class of analytical instruments and technological installations.

Keywords: electronic lens, focusing, equalization, aberration, sextupole, potential distribution.

In various devices and devices of electronic and ionic optics, significant aberrations of elements and assemblies of these devices often hinder the achievement of the required values of technical and operational characteristics. To eliminate such obstacles to the improvement of particle-beam systems, aberration correctors are used, which make it possible to significantly improve the technical characteristics of a wide class of analytical instruments and technological installations. Many works [1-14] are devoted to the problems of theoretical and applied research of the properties of electronic lenses and the search for possibilities for correcting aberrations. At present, the quadrupole components of focusing fields are most often used as aberration correctors. Sextupole correctors are also used, but they are used rather limitedly and their properties are poorly studied.

In this work, the possibility of correcting the aberrations of electronic lenses using the sextupole components of the fields is studied in sufficient detail. The equations for the trajectories of charged particles are derived and formulas are determined for the numerical calculation of the main parameters of a lens containing the axisymmetric and sextupole components of the fields.

Let us introduce a Cartesian coordinate system *x, y* and *z*, the *z* axis of which coincides with the main optical axis y of the lens under study.

The motion of charged particles in the studied electron lens is described by the following system of equations:

$$\ddot{x} = -\frac{e}{m}\frac{\partial\varphi}{\partial x},\ \ddot{y} = -\frac{e}{m}\frac{\partial\varphi}{\partial y}, \quad (1)$$

$$\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = -\frac{2e}{m}(\varphi+\varepsilon). \quad (2)$$

where, *e* and *m* are the charge and mass of the particle, $\varphi = \varphi\ (x, y, z)$ is the distribution of the electrostatic potential, the points denote differentiation in time, ε is the spread of the energy of charged particles in the beam.

Consider a variant of a cathode lens containing axisymmetric and sextupole components of the focusing potential distribution.

For a cathode lens, the initial conditions for equations (1), (2) have the form:

$$x(t)\,|_{t=0} = x_k,\ \ y(t)\,|_{t=0} = y_k,\ \ z(t)\,|_{t=0} = z_k, \quad (3)$$

$$\dot{x}(t)\,|_{t=t_R} = \sqrt{-\frac{2e}{m}\varepsilon_X} = \sqrt{-\frac{2e}{m}\varepsilon}\sin\theta\cos\alpha, \quad (4)$$

$$\dot{y}(t)\,|_{t=0} = \sqrt{-\frac{2e}{m}\varepsilon_Y} = \sqrt{-\frac{2e}{m}\varepsilon}\sin\theta\sin\alpha, \quad (5)$$

$$\dot{z}(t)\,|_{t=0} = \sqrt{-\frac{2e}{m}\varepsilon_Z} = \sqrt{-\frac{2e}{m}\varepsilon}\cos\theta, \quad (6)$$

where θ is the angle between the direction of emission of the particle emitted by the cathode and the main optical axis, α is the angle between the projection of the vector of the initial velocity of the particle emitted by the cathode onto the *xy* plane and the *x* axis; the subscript "*k"* denotes the value of the quantity at t *= 0,* i.e. at the cathode.

The distribution of the electrostatic potential near the main optical axis of the lens can be represented as the following series:

$$\varphi(x,y,z) = \Phi(z) - \frac{\Phi''}{4}x^2 - \frac{\Phi''}{4}y^2 + \\ +\varphi_3 x^3 + 3f_3x^2y - 3\varphi_3xy^2 - f_3y^3 + \\ +\frac{\Phi^{IV}}{64}x^4 + \frac{\Phi^{IV}}{32}x^2y^2 + \frac{\Phi^{IV}}{64}y^4 + \ldots \quad (7)$$

where $\Phi(z) = \varphi(0,0,z)$; $\varphi_3,\ f_3$ are functions that characterize the sixth field components of the field, the dashes denote differentiation along the *z* coordinate.

Let us assume that the center of the cathode coincides with the origin of coordinates, the potential of the cathode is taken to be zero, i.e.

$$\varphi_k = \varphi(x_k, y_k, z_k) = 0\,. \quad (8)$$

Substituting the value $\varphi(x,y,z)$ of the function from (7) into the system of equations (1), (2), we obtain the equations of motion of a charged particle in the field under study.

Let us consider the motion of an arbitrary particle in the beam relative to the motion of the reference particle, calling the motion of this particle the reference motion. Let us take as a reference particle moving along the *z* axis and having zero initial energy (i.e., ε = 0). In this case, the support motion is described by the equation

$$\dot{z}_{on}^2 = -\frac{2e}{m}\Phi(z_{on}), \quad (9)$$

where the subscript "*on*" denotes that the value belongs to the reference motion.

From (9) we have

$$\dot{z}_{on} = \sqrt{-\frac{2e}{m}\Phi(z_{on})} \ . \quad (10)$$

The *z* coordinate of an arbitrary particle can be expressed in terms of the axial coordinate of the reference particle z_{op} as follows

$$z = z_{on} + D_z(z_{on}) \quad (11)$$

Here the function $D_z(z_{on})$ describes the total longitudinal aberration of the lens under study.

Substituting (10) and (11) into the equations of motion, we obtain

$$2\Phi x'' + \Phi' x' + \frac{\Phi''}{2}x = -\frac{\Phi'''}{2}D_z x - \frac{\Phi^{IV}}{4}D_z^2 x +$$
$$+3\varphi_3 x^2 + 3\varphi_3' D_z x^2 + 6 f_3 xy + 6 f_3' D_z xy -$$
$$-3\varphi_3 y^2 - 3\varphi_3' D_z y^2 + \frac{\Phi^{IV}}{16}x^3 + \frac{\Phi^{IV}}{16}xy^2, \quad (12)$$

$$2\Phi y'' + \Phi' y' + \frac{\Phi''}{2}y = -\frac{\Phi'''}{2}D_z y - \frac{\Phi^{IV}}{4}D_z^2 y +$$
$$-3 f_3 y^2 - 3 f_3' D_z y^2 - 6\varphi_3 xy - 6\varphi_3' D_z xy +$$
$$+3 f_3 x^2 + 3 f_3' D_z x^2 + \frac{\Phi^{IV}}{16}y^3 + \frac{\Phi^{IV}}{16}x^2 y, \quad (13)$$

$$2\Phi D_z' - \Phi' D_z = -\Phi x'^2 - \Phi y'^2 -$$
$$-\Phi D_z'^2 + \frac{\Phi''}{2}D_z^2 - \frac{\Phi''}{4}x^2 - \frac{\Phi''}{4}y^2 + \varepsilon. \quad (14)$$

In equations (12) - (14) and further, until it is specifically stated, the arguments of all functions are $z_{ОП}$, and the primes denote differentiation with respect to $z_{ОП}$. When deriving the last equations, we used expansions of the form

$$\Phi(z) = \Phi(z_{ОП} + D_z) = \Phi(z_{ОП}) + \Phi'(z_{ОП})D_z(z_{ОП}) + \frac{\Phi''(z_{ОП})}{2}D_Z^2(z_{ОП}) + \ldots \quad (15)$$

Equations (12) - (14) are equations of motion of charged particles in parametric form. The coordinate of the reference particle is taken as a parameter. Note that on the right-hand sides of the equations of motion, terms are retained not higher than the third order of smallness.

To derive the equations of trajectories in parametric form, it is necessary to solve the system of equations (12) - (14). Equations (12) and (13) are linear nonhomogeneous differential equations of the second order, and equation (14) is a linear nonhomogeneous equation of the first order.

The system of equations (12) - (14) can be solved by the method of successive approximations. In the first approximation, we find solutions to linear homogeneous equations, therefore, we take equal to zero the right-hand sides of these equations, then they will take the form:

$$2\Phi x_1'' + \Phi' x_1' + \frac{\Phi''}{2}x_1 = 0, \quad (16)$$

$$2\Phi y_1'' + \Phi' y_1' + \frac{\Phi''}{2}y_1 = 0, \quad (17)$$

$$2\Phi D_{Z1}' - \Phi' D_{Z1} = 0. \quad (18)$$

Index "1" means that the values of the quantities are determined in the first approximation.

Taking into account condition (6), the solution to equation (18) will be

$$D_{Z1} = \frac{2}{\Phi_K'}\sqrt{\Phi}\sqrt{\varepsilon_z}. \quad (19)$$

From (19) it is seen that D_{z1} is a quantity of the first order of smallness. General solutions of linear homogeneous differential equations of the second order (16) and (17) have the form:

$$x_1 = a_x U_x + b_x V_x, \ y_1 = a_y U_y + b_y V_y, \quad (20)$$

where a_x, b_x, a_y and b_y *are* arbitrary constants determined from the initial conditions; U_x, V_x, U_y, and V_y are particular linearly independent solutions of equations (16) and (17).

Using the initial conditions, one can see that the particular linearly independent solutions U_x, V_x, U_y and V_y are related to each other by the following expression

$$\sqrt{\Phi}\left(U_x V_x' - U_x' V_x\right) = \sqrt{\Phi}\left(U_y V_y' - U_y' V_y\right) = \frac{\Phi_K'}{2}. \quad (21)$$

General solutions of the (12) - (14) equations in the form:

$$x = x_1 + D_{x2} + D_{x3}, \ y = y_1 + D_{y2} + D_{y3}, \quad (22)$$

$$D_z = D_{z1} + D_{z2}. \quad (23)$$

The numbers 1, 2 and 3 in the indices indicate the order of smallness of the corresponding values.

In equations (22) and (23), respectively, D_{x2}, D_{x3}, D_{y2} and D_{y3} are vertical aberrations of the second and third orders, D_{z1} and D_{z2} are longitudinal aberrations of the first and second orders of smallness.

Since, equation (16) and (17) are of a singular point at $z = z_k$, which are the poles of the first kind, solutions V_x and V_y can be represented as:

$$V_x = W_x\sqrt{\Phi}, \ V_y = W_y\sqrt{\Phi}. \quad (24)$$

W_x and W_y are analytical functions that satisfy the equations:

$$2\Phi W_x'' + 3\Phi' W_x' + \frac{3}{2}\Phi'' W_x = 0, \quad (25)$$

$$2\Phi W_y'' + 3\Phi' W_y' + \frac{3}{2}\Phi'' W_y = 0. \quad (26)$$

Equations (16), (17), (25), (26) are solved under the following initial conditions:

$$U_x(z_{on})|_{z_{on}=0} = U_y(z_{on})|_{z_{on}=0} = W_x(z_{on})|_{z_{on}=0} = W_y(z_{on})|_{z_{on}=0} = 1, \quad (27)$$

$$U_x'(z_{on})|_{z_{on}=0} = U_y'(z_{on})|_{z_{on}=0} = \frac{1}{R}, \quad (28)$$

$$W_x^{'}(z_{on})|_{z_{on}=0}=W_y^{'}(z_{on})|_{z_{on}=0}=\frac{1}{R}, \quad (29)$$

$$R=-\frac{\varPhi_K^{''}}{2\varPhi_K^{'}}. \quad (30)$$

R_x and R_y *are the* radii of curvature of the cathode surface in the horizontal (*xz*) and vertical (*yz*) directions.

Arbitrary constants a_x, a_y, b_x and b_y for solutions (20) are determined from the analysis of equations (16), (17), (27) - (30) and initial conditions (3) - (6)

$$a_x = x_K,\ a_y = y_K, \quad (31)$$

$$b_x=\frac{2}{\varPhi_K^{'}}\sqrt{\varepsilon_x},\ b_y=\frac{2}{\varPhi_K^{'}}\sqrt{\varepsilon_y}. \quad (32)$$

Substituting (31) and (32) in (20) and (12) - (14), we obtain several equations characterizing the set of aberration coefficients inherent in the studied electron lens. By varying the focusing field parameters, specific lenses can be selected to correct a range of aberrations. In practice, the sextupole components of focusing fields can be used quite successfully for a noticeable decrease in chromatic aberrations of the second order of smallness.

(11), (22) and (23) are the equations of trajectories in parametric form for the lens under study. Mathematical expressions and formulas for the analysis of the paraxial properties and aberration characteristics of the investigated element are determined by solution (12) - (14) by the method of successive approximations considering the above initial conditions.

When calculating the spatial and time-of-flight parameters of the lens, the transition to an explicit dependence on the coordinate of the main optical axis is performed considering (11). Solving equation (11) by the method of successive approximations, we express z_{on} through the coordinate of the main optical axis

$$z_{on}=z-\zeta_1(z)-\zeta_2(z)+\zeta_1^{'}(z)\zeta_1(z). \quad (33)$$

Substituting (33) into equations (22), we find the equations of the trajectories of charged particles in an explicit dependence on the coordinate z

$$x(z)=x_1(z)-x_1^{'}(z)\left[\zeta_1(z)+\zeta_2(z)-\zeta_1^{'}(z)\zeta_1(z)\right]+\frac{1}{2}x_1^{''}(z)\zeta_1^2(z)+D_{x2}(z)+D_{x3}(z), \quad (34)$$

$$y(z)=y_1(z)-y_1^{'}(z)\left[\zeta_1(z)+\zeta_2(z)-\zeta_1^{'}(z)\zeta_1(z)\right]+\frac{1}{2}y_1^{''}(z)\zeta_1^2(z)+D_{y2}(z)+D_{y3}(z). \quad (35)$$

Equations (34) and (35) characterize the spatial focusing of charged particles in the lens under study.

Time-of-flight focusing in the lenses under consideration is described by the equation

$$t(z)=\sqrt{-\frac{m}{2e}}\left\{\int\frac{dz}{\sqrt{\Phi}}-\frac{1}{\sqrt{\Phi}}\left[\zeta_1(z)+\zeta_2(z)-\zeta_1^{'}(z)\zeta_1(z)\right]\right\}. \quad (36)$$

Specific expressions for determining the entire set of aberration coefficients are rather cumbersome and are not presented here.

We note that the general expressions obtained in this work can be used to carry out detailed studies of the focusing properties of specific electro-optical elements and to find the most effective designs of charged particle sources with correction for second and third order aberrations.

References

1. Kel`man, V. M., Yavor, S. Ya. [Electron optics]. AN SSSR, Moscow-Leningrad, 1963.
2. Glaser, W. (1952). Grundlagen der Electronenoptik. Vienna: Springer-Verlag. (Russian transl.: GIITL, Moscow 1957).
3. Szilagyi, M., Electron and Ion Optics. Plenum Publishing Corporation, New York, 1988.
4. Ibrayev, A.T. Theory of Cathode Lens with Multipole Components of Electrostatic Field and the Space Charge. doi:10.1017/S1431927615013495. – Microscopy and Microanalysis, 2015, vol. 21, N6, 270-275 p.
5. Sherstnev, L.G. [Electronic optics and electron-beam tubes]. Moscow, Energiya, 1971.
6. Bonshtedt, B. E. [Measuring aberrations of a cathode lens]. R and E, 1964, vol. 9, 844-850 p.
7. Vlasov, A.G., Shapiro, Yu.A. [Methods for Calculation of Emission electron-optical systems]. Mashinostroenie, Leningrad, 1974, 183 p.
8. V.P. Il'in, V.A. Kateshov, Yu.V. Kulikov, M.A. Monastyrsky. Emission-Imaging Electron-Optical System Design. doi:10.1016/S0065-2539(08)60390-X. – Novosibirsk, Nauka, 1987.
9. Flegontov Yu.A., Zolina N.K. [On the calculation of Features of Electronic Image in Electrostatic Cathode Lenses]. Zhurn. Tekhn. Fiz.,1978, vol. 48, №12, 2479-2489 p.
10. Abdulmanov V.G. Anode modulator of electron-beam sources of multicharged ions. doi: 10.1117/12.511993. – Proceedings SPIE., 2003, vol. 5025, 89-97 p.
11. Kel`man, V. M., Sapargaliev, A. A., Yakushev, E. M. Theory of the Cathode lenses. Zhurnal Tekhnicheskoi Fiziki, 1974, 44, 928–943 p.
12. Ibrayev, A. T., Sapargaliev, A. A. Transaxial electrostatic cathode lens. Zhurnal Tekhnicheskoi Fiziki, 1981, 51, 22–30 p.
13. Yakushev, E. Theory and Computation of Electron Mirrors: The Central Particle Method. Advances in Imaging and Electron Physics. doi:10.1016/B978-0-12-407701-0.00003-0. – First Edition, 2013, 147–247 p.
14. Ibrayev, A. T., Sagyndyk A. B. (2017). Numerical investigation of the aberrational coefficients of a box shaped cathode lens. News of the national academy of sciences of the republic of Kazakhstan. Series of geology and technical sciences. ISSN 2224-5278. Volume 4, Number 424, 108 — 114.